

Capítulo 6

Centro de gravedad, Centro de masa y Centroide

Estática 2015-1

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Introducción

6.1 Centro de gravedad y Centro de masa para un sistema de partículas.

6.2 Centro de gravedad, Centro de masa y Centroide para un cuerpo.

6.3 Cuerpos compuestos.

6.4 Posicionamiento del CM en cuerpos articulados en un punto.

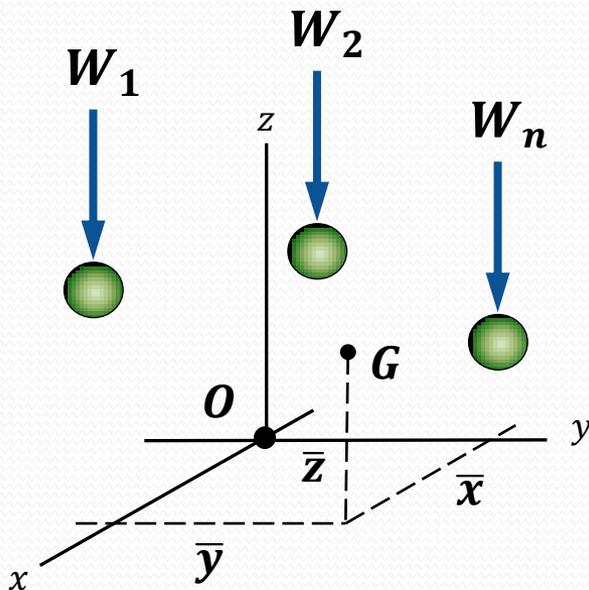
6.1

Centro de gravedad y Centro de masa para un sistema de partículas.

6.1 Centro de gravedad y Centro de masa para un sistema de partículas.

Centro de gravedad

Es un punto donde se ubica el **peso resultante**



$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W}$$

$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W}$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W}$$

6.1 Centro de gravedad y Centro de masa para un sistema de partículas.

Centro de masa

Si la aceleración debido a la gravedad es constante, entonces $\mathbf{W} = \mathbf{m} \cdot \mathbf{g}$

$$\bar{x} = \frac{\sum \tilde{x} m}{\sum m} \quad \bar{y} = \frac{\sum \tilde{y} m}{\sum m} \quad \bar{z} = \frac{\sum \tilde{z} m}{\sum m}$$

6.2

Centro de gravedad, Centro de masa y Centroide para un cuerpo.

6.2 Centro de gravedad, Centro de masa y Centroide para un cuerpo.

Centro de gravedad

$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W}$$

$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W}$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W}$$

$$\bar{x} = \frac{\int \tilde{x} dW}{\int dW}$$

$$\bar{y} = \frac{\int \tilde{y} dW}{\int dW}$$

$$\bar{z} = \frac{\int \tilde{z} dW}{\int dW}$$

6.2 Centro de gravedad, Centro de masa y Centroide para un cuerpo.

Centro de masa

El diferencial de dW debe ser expresado en términos de su volumen dV

$$dW = \gamma dV$$

γ : peso específico [N/m^3]

$\gamma = \rho g$: densidad . gravedad [kg/m^3].
[m/s^2]

$$\bar{x} = \frac{\int \tilde{x} \rho g dV}{\int \rho g dV} \quad \bar{y} = \frac{\int \tilde{y} \rho g dV}{\int \rho g dV} \quad \bar{z} = \frac{\int \tilde{z} \rho g dV}{\int \rho g dV}$$

6.2 Centro de gravedad, Centro de masa y Centroide para un cuerpo.

Centroide

- Es el **centro geométrico** de un cuerpo
- Si el material del cuerpo es uniforme u homogéneo, el **peso específico o la densidad serán constante**, entonces se puede utilizar las expresiones usadas para determinar el centro de masa.

$$\gamma, \rho = \textit{constante}$$

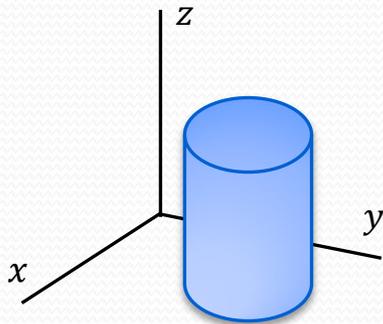
$$\bar{x} = \frac{\int \tilde{x} \rho g dV}{\int \rho g dV}$$

6.2 Centro de gravedad, Centro de masa y Centroide para un cuerpo.

Centroide

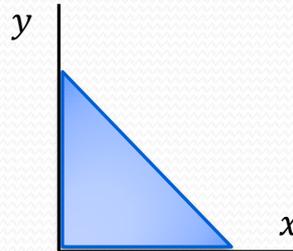
$$\bar{x} = \frac{\int \tilde{x} dV}{\int dV}$$

Volumen



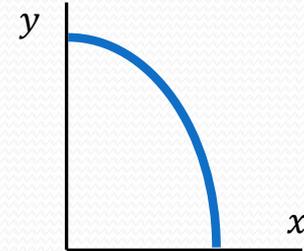
$$\bar{x} = \frac{\int \tilde{x} dA}{\int dA}$$

Área



$$\bar{x} = \frac{\int \tilde{x} dL}{\int dL}$$

Línea

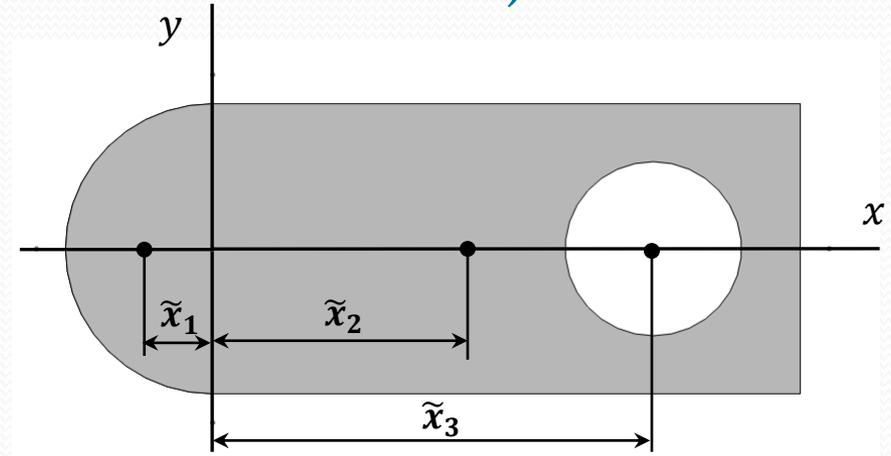
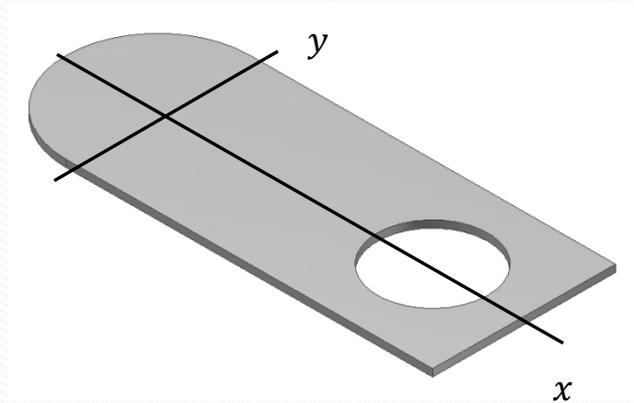


6.3

Cuerpos compuestos

6.3 Cuerpos compuestos.

Área compuesta (Centroide = Centro de masa)



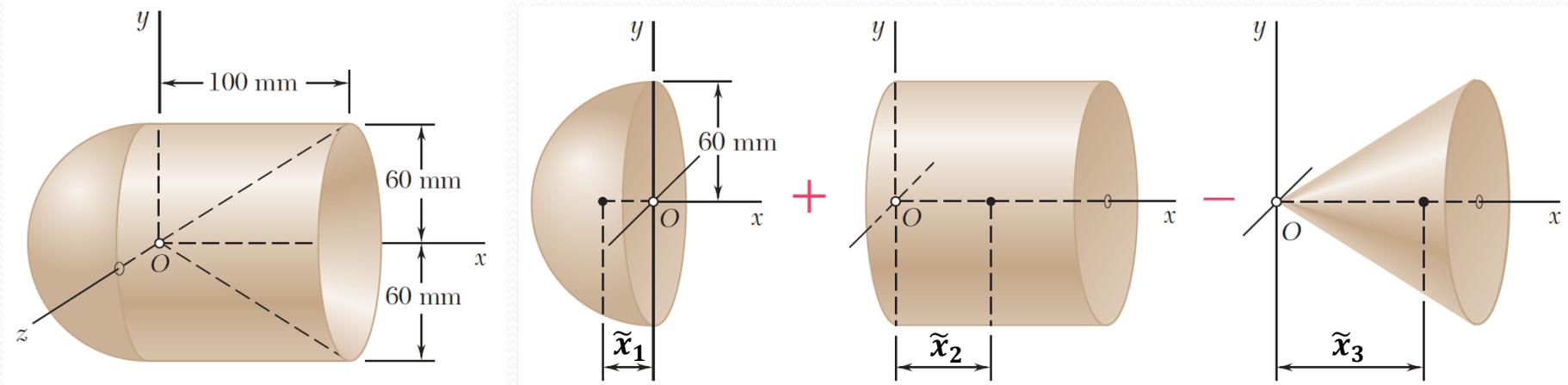
	A_i	\tilde{x}_i	\tilde{y}_i	$\tilde{x}_i A_i$	$\tilde{y}_i A_i$
Semi círculo	(+)	(-)	0	(+)(-) = (-)	0
Rectángulo	(+)	(+)	0	(+)(+) = (+)	0
Círculo	(-)	(+)	0	(-)(+) = (-)	0
Total	(+)	---	---	(?)	0

$$\bar{x} = \frac{\sum \tilde{x}_i A_i}{\sum A_i}$$

$$\bar{y} = \frac{\sum \tilde{y}_i A_i}{\sum A_i} = 0$$

6.3 Cuerpos compuestos.

Volumen compuesto (Centroide = Centro de masa)



	V_i	\tilde{x}_i	\tilde{y}_i	\tilde{z}_i	$\tilde{x}_i V_i$	$\tilde{y}_i V_i$	$\tilde{z}_i V_i$
Semi esfera	(+)	(-)	0	0	(+)(-) = (-)	0	0
Cilindro	(+)	(+)	0	0	(+)(+) = (+)	0	0
Cono	(-)	(+)	0	0	(-)(+) = (-)	0	0
Total	(+)	---	---	---	(?)	0	0

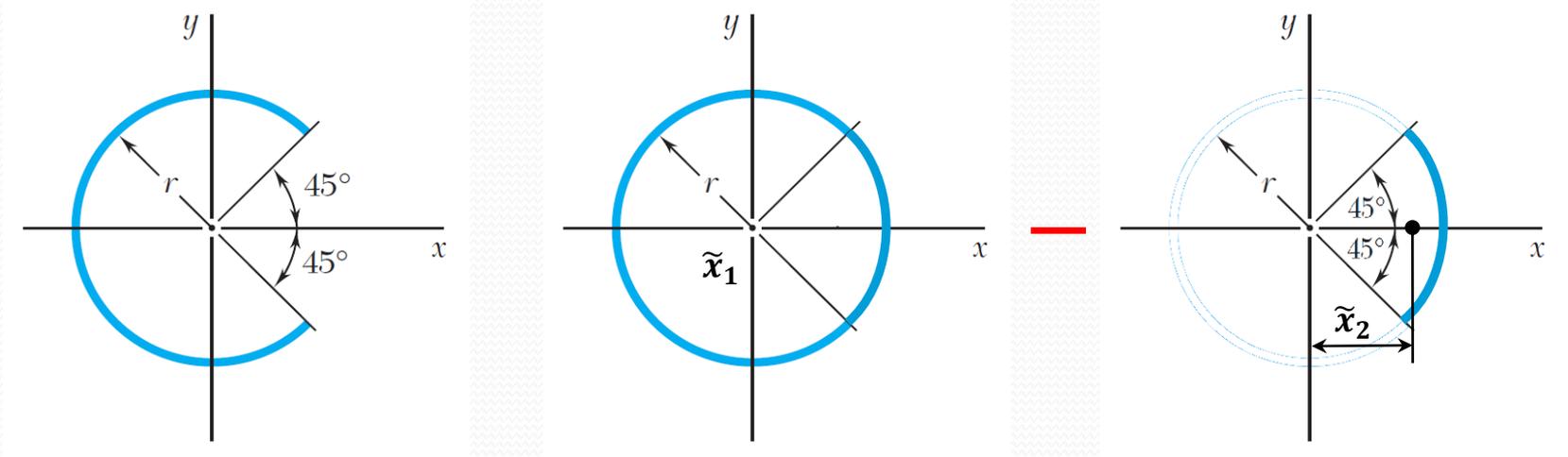
$$\bar{x} = \frac{\sum \tilde{x}_i V_i}{\sum V_i}$$

$$\bar{y} = \frac{\sum \tilde{y}_i V_i}{\sum V_i} = 0$$

$$\bar{z} = \frac{\sum \tilde{z}_i V_i}{\sum V_i} = 0$$

6.3 Cuerpos compuestos.

Línea compuesta (Centroide = Centro de masa)



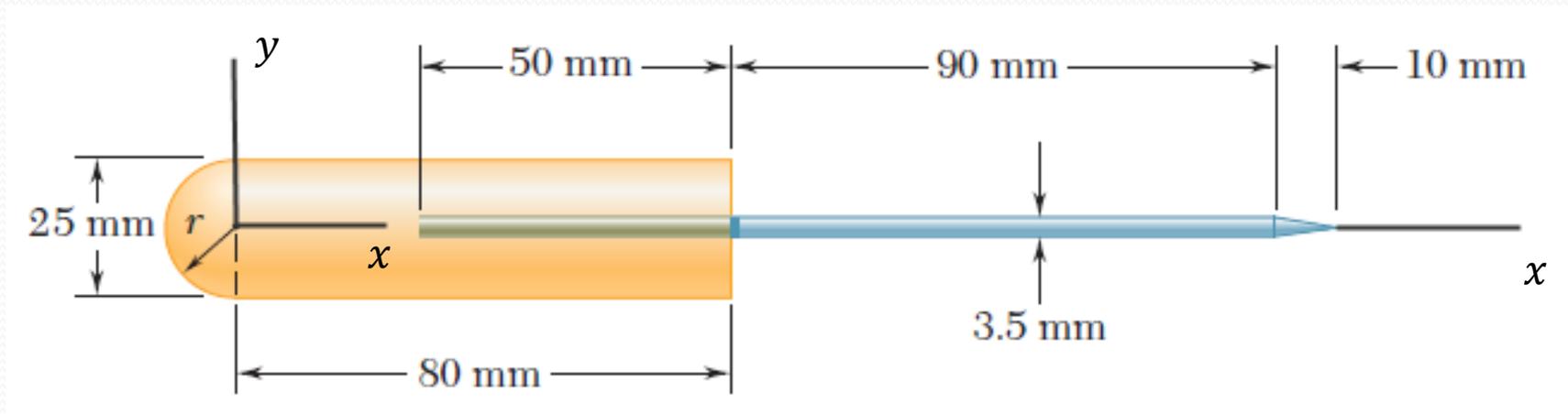
	L_i	\tilde{x}_i	\tilde{y}_i	$\tilde{x}_i L_i$	$\tilde{y}_i L_i$
Circunferencia completa	(+)	0	0	0	0
Arco de circunferencia	(-)	(+)	0	(+)(+) = (-)	0
Total	(+)	---	---	(-)	0

$$\bar{x} = \frac{\sum \tilde{x}_i L_i}{\sum L_i}$$

$$\bar{y} = \frac{\sum \tilde{y}_i L_i}{\sum L_i} = 0$$

6.3 Cuerpos compuestos.

Cuerpo compuesto de diferentes densidades (Centro de masa)



	V_i	$\gamma_i = \rho_i \cdot g$	W_i	\tilde{x}_i	\tilde{y}_i	\tilde{z}_i	$\tilde{x}_i W_i$	$\tilde{y}_i W_i$	$\tilde{z}_i W_i$
Mango de plástico		$\gamma_{plástico}$			0	0		0	0
Aguja de acero		γ_{acero}			0	0		0	0
Total				---	---	---		0	0

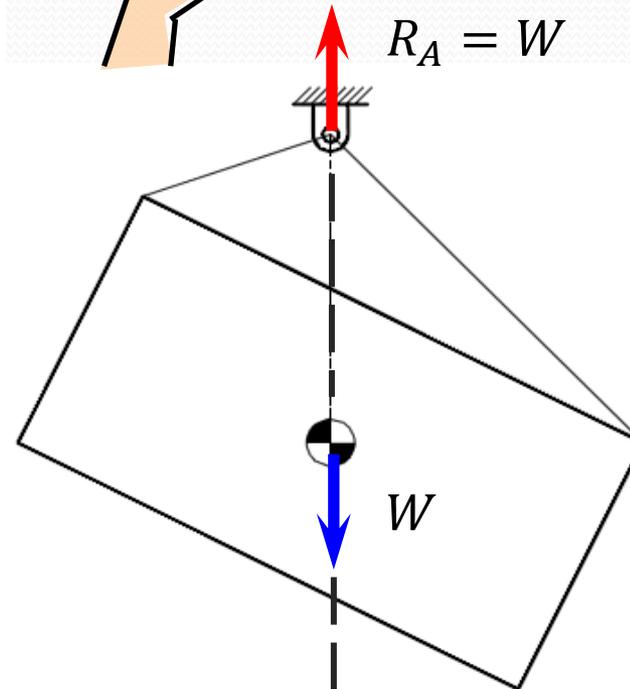
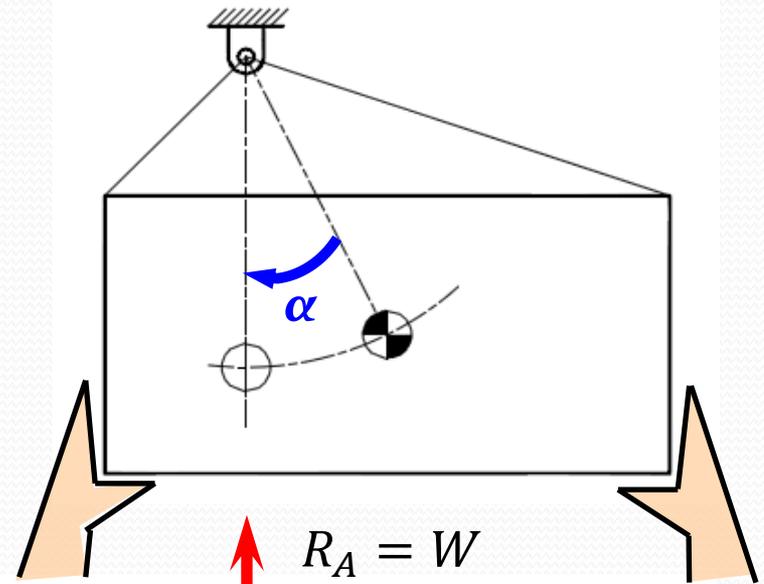
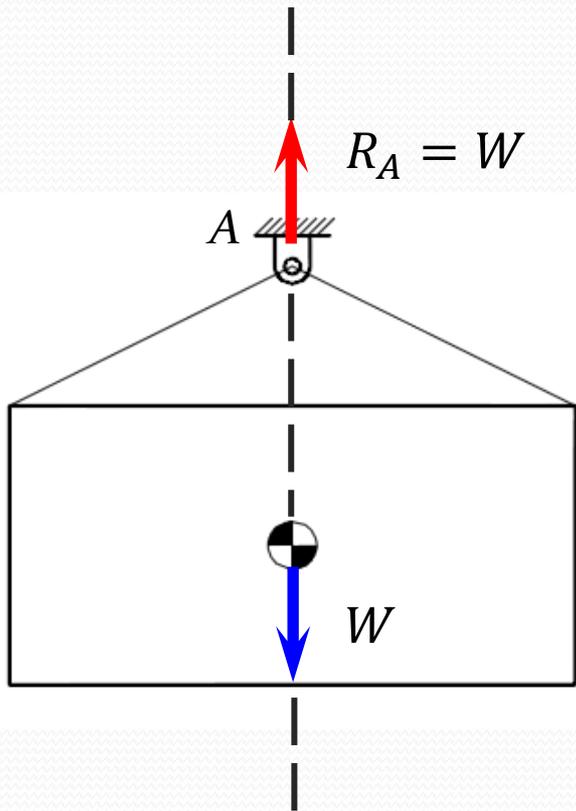
$$\bar{x} = \frac{\sum \tilde{x}_i W_i}{\sum W_i}$$

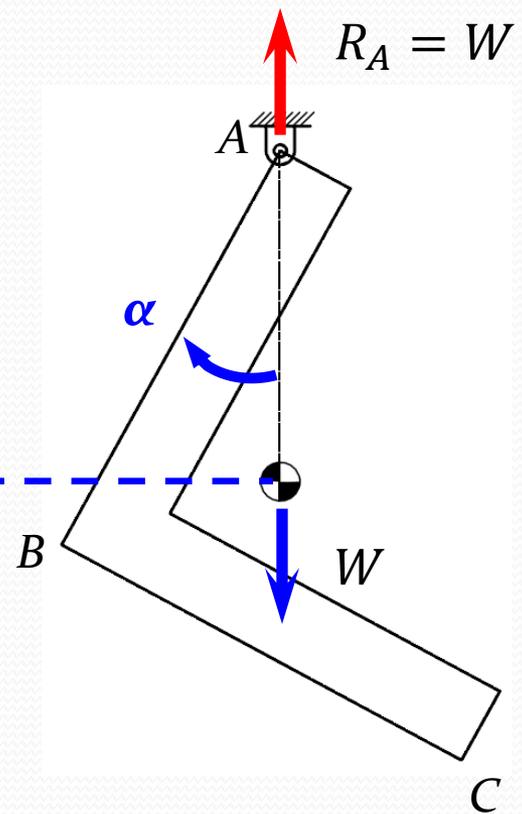
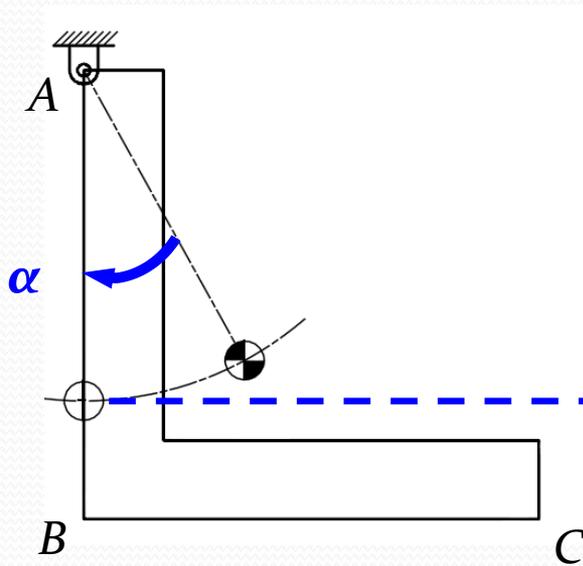
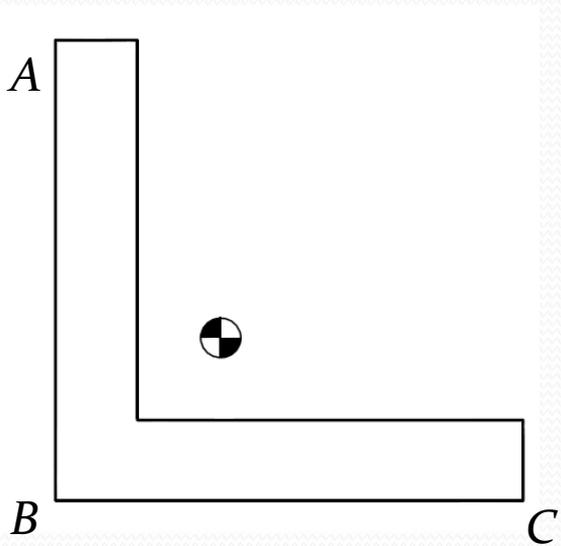
$$\bar{y} = \frac{\sum \tilde{y}_i W_i}{\sum W_i} = 0$$

$$\bar{z} = \frac{\sum \tilde{z}_i W_i}{\sum W_i} = 0$$

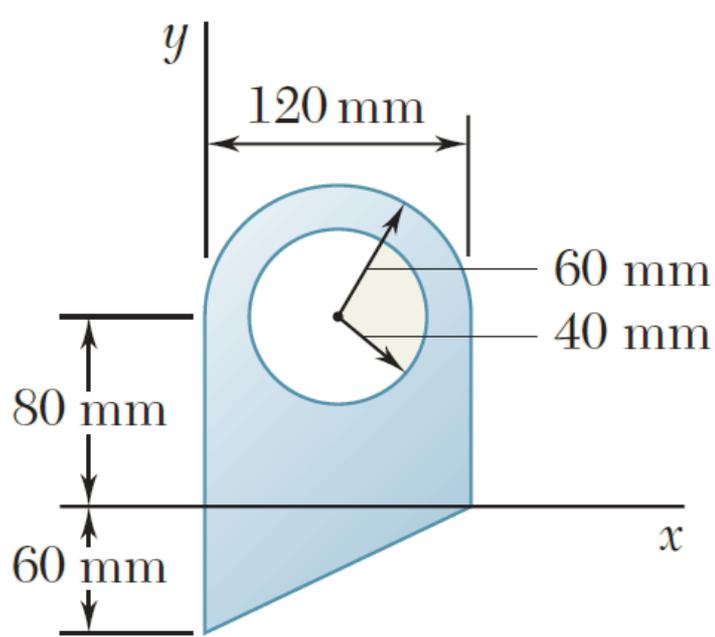
6.3

Posicionamiento del CM en cuerpos articulados en un punto.





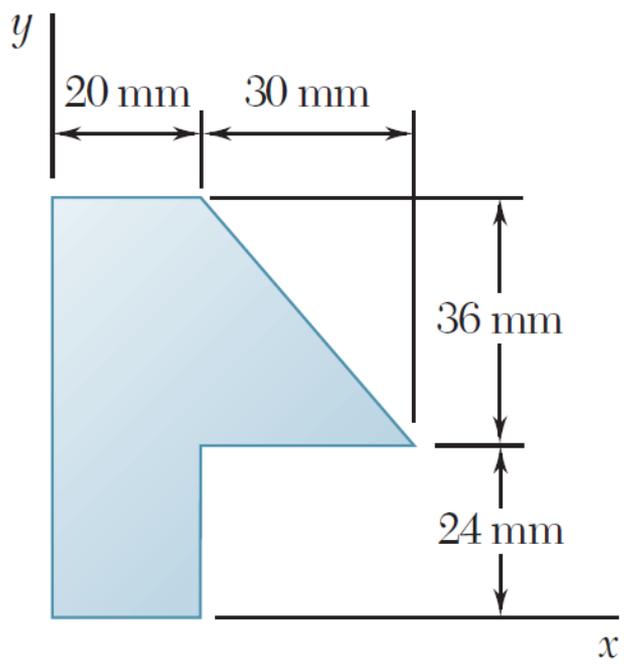
Ejercicios propuestos



Componente	A	\tilde{x}	\tilde{y}	$\tilde{x}A$	$\tilde{y}A$
Rectángulo	=120x80				
Semicírculo		=120/2			
Triángulo			=-60/3		
Círculo	= - $\pi 40^2$				
Total					

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = 54.79$$

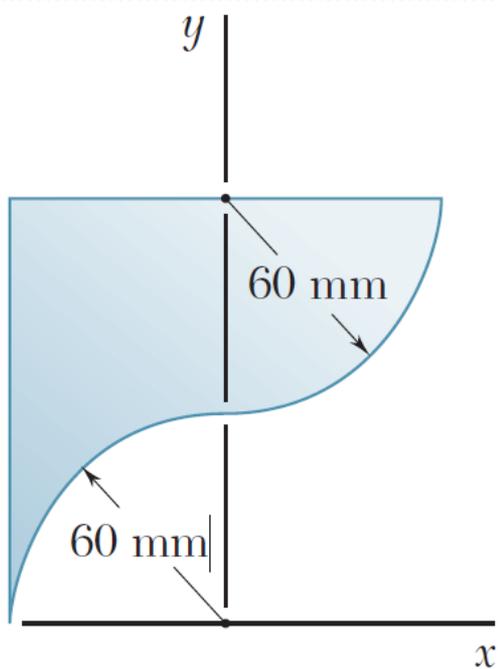
$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} =$$



Componente	A	\tilde{x}	\tilde{y}	$\tilde{x}A$	$\tilde{y}A$
Rectángulo	=20x60				
Triángulo		=20+30/3			
Total					

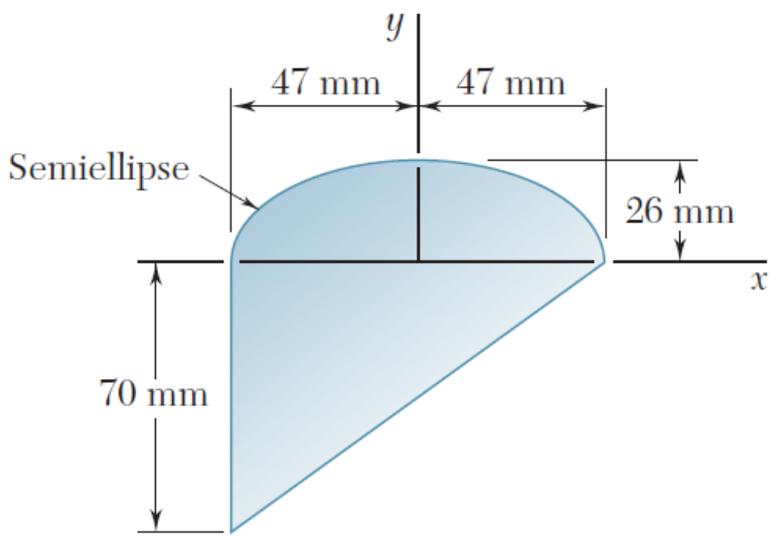
$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = 16.21$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} =$$



Componente	A	\tilde{x}	\tilde{y}	$\tilde{x}A$	$\tilde{y}A$
Rectángulo	$=60 \times 120$				
Cuarto Circ		$=4 \times 60 / 3\pi$			
Cuarto Circ			$=4 \times 60 / 3\pi$		
Total					

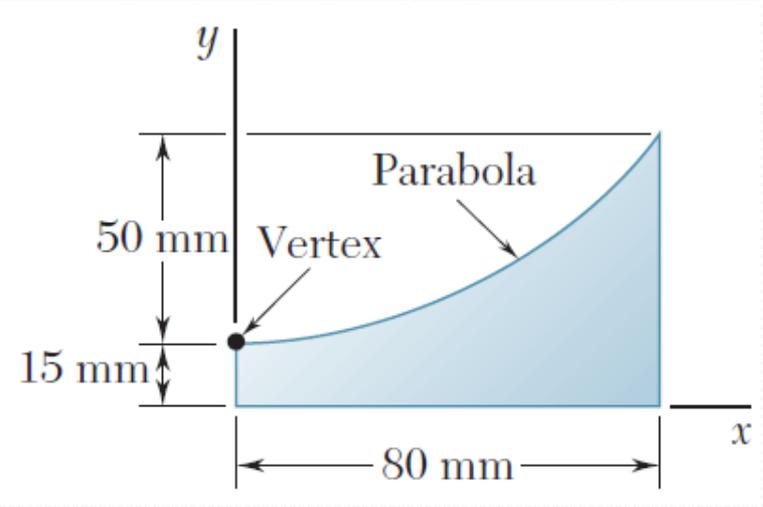
$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \quad -10.00 \quad \bar{y} = \frac{\sum \tilde{y}A}{\sum A} =$$



Componente	A	\tilde{x}	\tilde{y}	$\tilde{x}A$	$\tilde{y}A$
Semiellipse		=0			
Triángulo			=-70/3		
Total					

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \quad -9.90$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} =$$

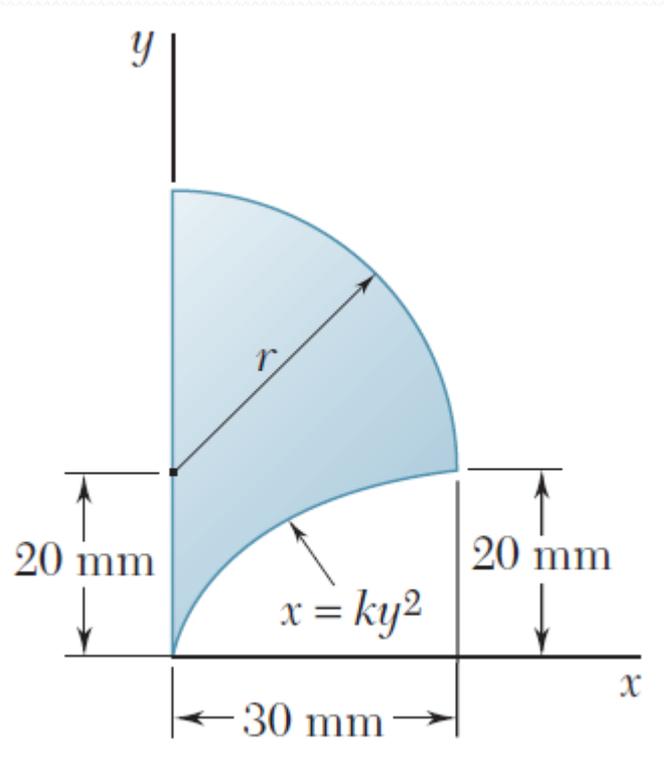


Componente	A	\tilde{x}	\tilde{y}	$\tilde{x}A$	$\tilde{y}A$
Rectángulo		=80/2			
Semiparaból.					
Total					

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A}$$

50.53

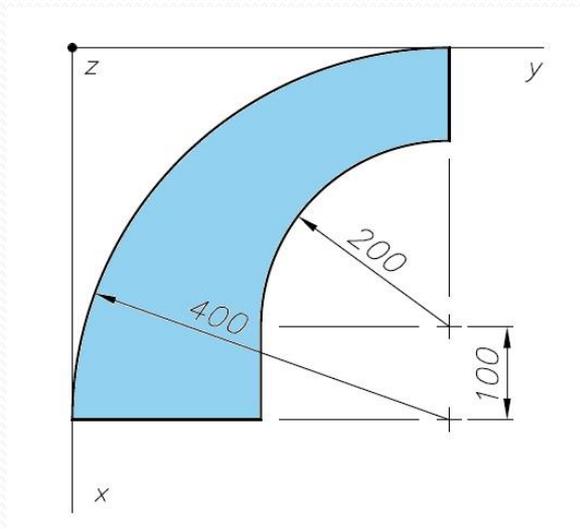
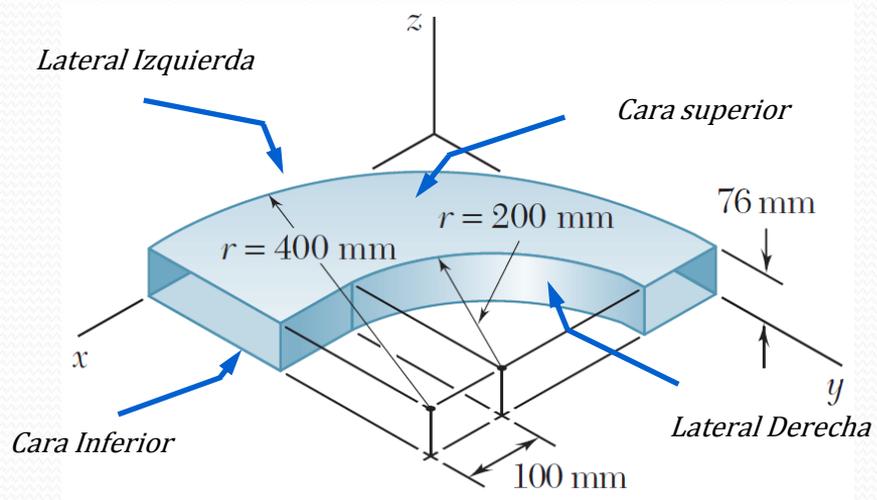
$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} =$$



Componente	A	\tilde{x}	\tilde{y}	$\tilde{x}A$	$\tilde{y}A$
Cuarto Circ	$=\pi 30^2 / 4$				
Enjunta Parb					
Total					

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} \quad \mathbf{11.91}$$

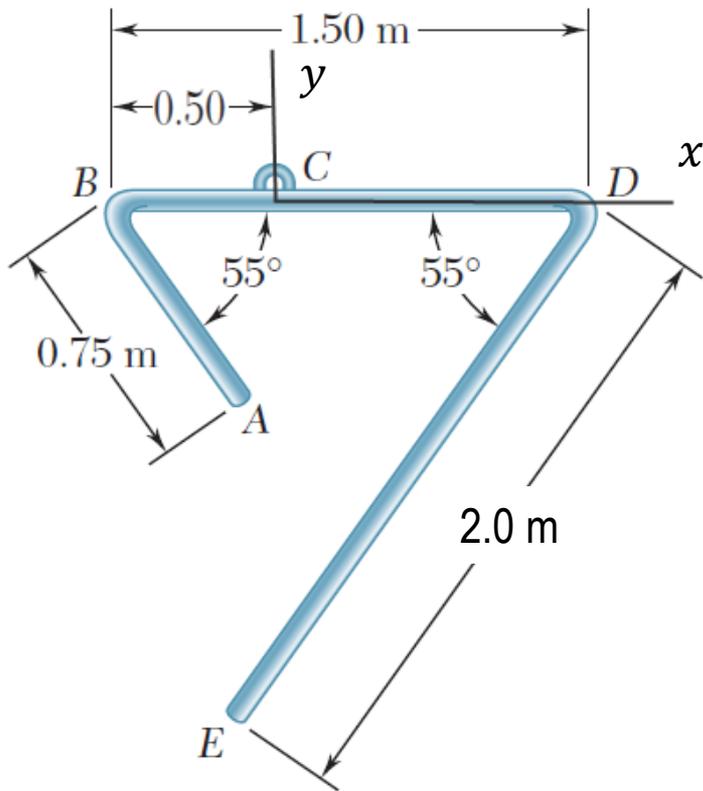
$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} =$$



	Componente	A	\tilde{x}	\tilde{y}	$\tilde{x}A$	$\tilde{y}A$
Cara Sup	1/4 Circu 400	$=\pi 400^2 / 4$				
			230.235			
	1/4 Circu 200			$=400 - 4 \times 200 / 3\pi$		-9899703.95
	Rectg 2x1				-7000000	-6000000
Cara Inf	1/4 Circu 400					
	1/4 Circu 200					
	Rectg 2x1					
L Izq	Sup R 400					6940883.334
Lat Der.	Sup R 200					6510441.667
	Sup 100x76					1520000
	Total	227723.87				

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = 193.53 \quad \bar{y} = \frac{\sum \tilde{y}A}{\sum A} = 180.20 \quad \bar{z} = \frac{\sum \tilde{z}A}{\sum A} =$$

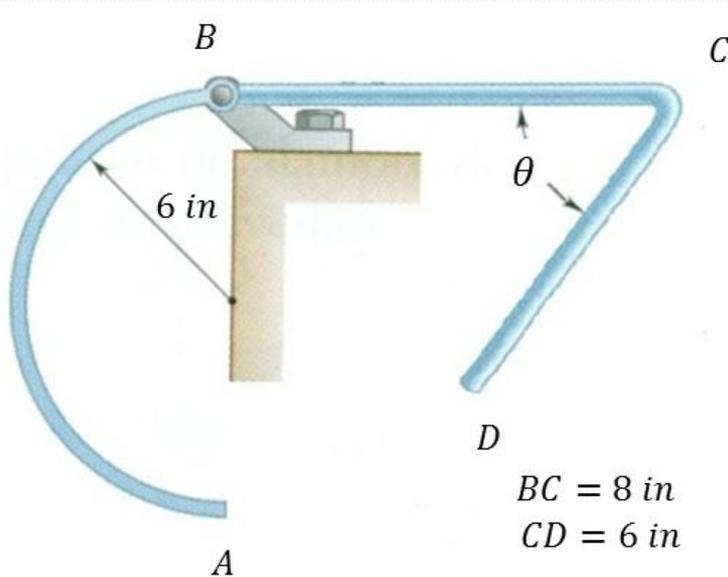
Si se cuelga el conjunto desde el punto C, determinar el ángulo que formaría el segmento CD con la horizontal



Segmento	L	\tilde{x}	\tilde{y}	$\tilde{x}L$	$\tilde{y}L$
AB					
BD					
DE					
Total					

$$\bar{x} = \frac{\sum \tilde{x}L}{\sum L} = 0.2386 \quad \bar{y} = \frac{\sum \tilde{y}L}{\sum L} =$$

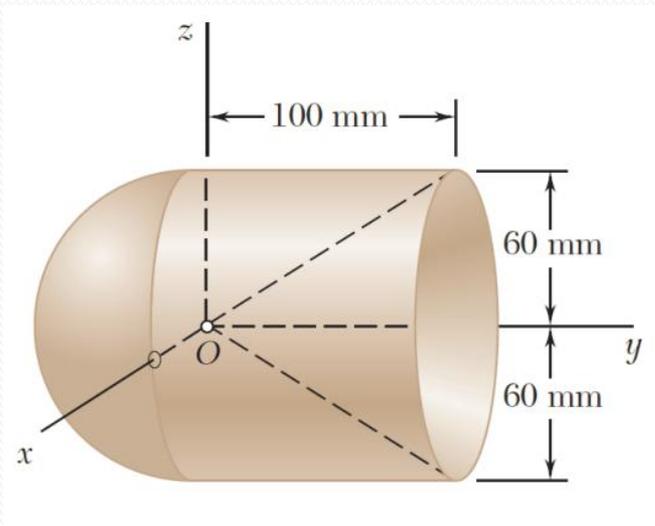
Determinar el ángulo tal que el conjunto permanezca en la posición mostrada.



Segmento	L	\tilde{x}	\tilde{y}	$\tilde{x}L$	$\tilde{y}L$
AB					
BC					
CD					
Total					

$$\bar{x} = \frac{\sum \tilde{x}L}{\sum L} =$$

$$\bar{y} = \frac{\sum \tilde{y}L}{\sum L} =$$

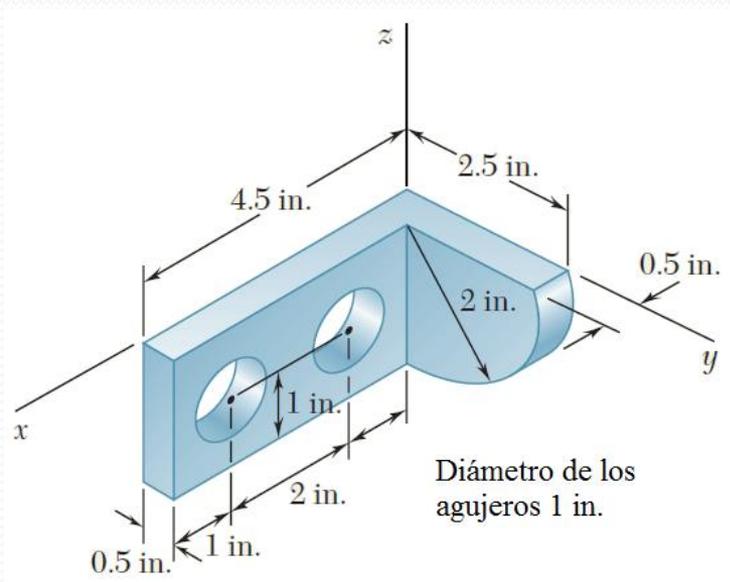


Componente	V	\tilde{x}	\tilde{y}	\tilde{z}	$\tilde{x}V$	$\tilde{y}V$	$\tilde{z}V$
Semi esfera	452160						
Cilindro							
Cono							
Total	1205760						

$$\bar{x} = \frac{\sum \tilde{x}V}{\sum V} = 0$$

$$\bar{y} = \frac{\sum \tilde{y}V}{\sum V} = 15$$

$$\bar{z} = \frac{\sum \tilde{z}V}{\sum V} =$$

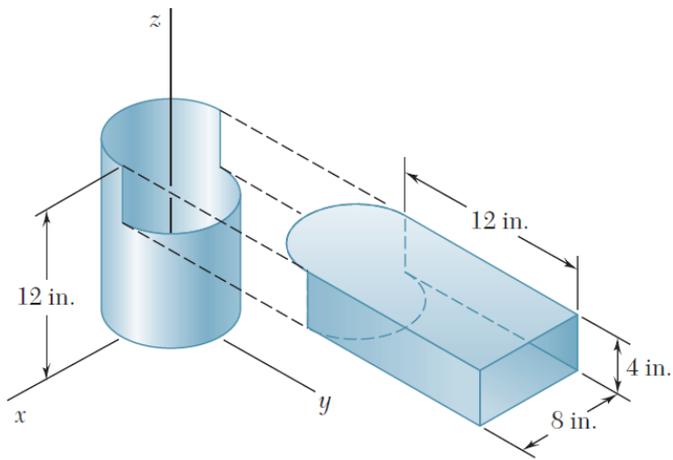


Componente	V	\tilde{x}	\tilde{y}	\tilde{z}	$\tilde{x}V$	$\tilde{y}V$	$\tilde{z}V$
PL Rect.	4.5						
PL Cuarto Circ	1.57						
Agujero 1							
Agujero 2							
Total	5.285						

$$\bar{x} = \frac{\sum \tilde{x}V}{\sum V} = 1.62$$

$$\bar{y} = \frac{\sum \tilde{y}V}{\sum V} =$$

$$\bar{z} = \frac{\sum \tilde{z}V}{\sum V} =$$

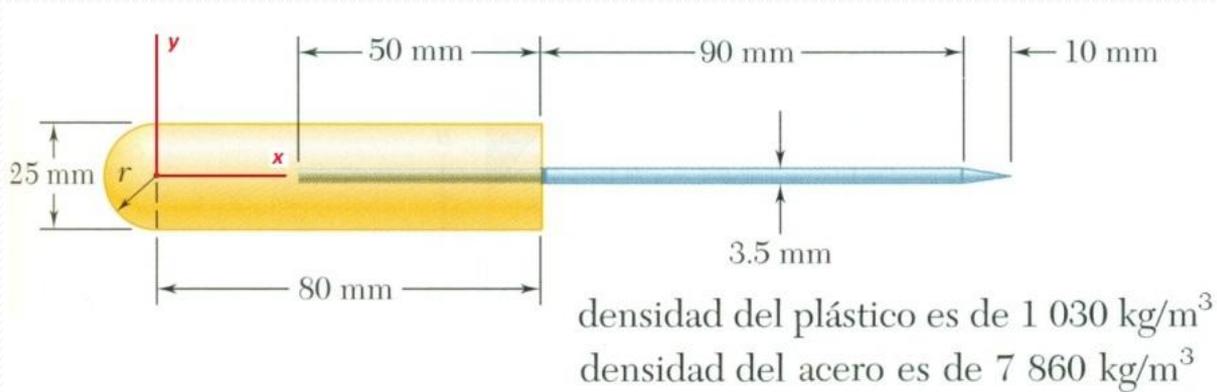


Componente	A	\tilde{x}	\tilde{y}	\tilde{z}	$\tilde{x}A$	$\tilde{y}A$	$\tilde{z}A$
Ducto Cilind	200.96						
Ducto S-Cilind	50.24						
Ducto Rectang							
Semi círculo 1							
Semi círculo 2	-25.12						
Total	539.20						

$$\bar{x} = \frac{\sum \tilde{x}A}{\sum A} =$$

$$\bar{y} = \frac{\sum \tilde{y}A}{\sum A} = 2.809$$

$$\bar{z} = \frac{\sum \tilde{z}A}{\sum A} =$$



Componente	V	$\rho \cdot g$	W	\tilde{x}	\tilde{y}	\tilde{z}	$\tilde{x}W$	$\tilde{y}W$	$\tilde{z}W$
Semi esf. Plast.		1.010E-05							
Cilindro Plast.									
Cilindro Plast.									
Cilindro Acero									
Cono Acero									
Total									

$$\bar{x} = \frac{\sum \tilde{x}W}{\sum W} = 48.6$$

$$\bar{y} = \frac{\sum \tilde{y}W}{\sum W} =$$

$$\bar{z} = \frac{\sum \tilde{z}W}{\sum W} =$$